AP Calculus AB

Unit 6 Review

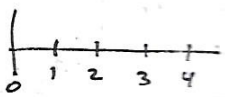
1) $\frac{1}{\pi/4}\int_0^{\pi/4} \sin x\,dx$

$\frac{4}{\pi}\left[-\cos x + C\right]\Big|_0^{\pi/4}$

$\frac{4}{\pi}\left[-\cos\frac{\pi}{4} + \cos 0\right]$

$\frac{4}{\pi}\left[-\frac{\sqrt{2}}{2} + 1\right]$

2) $\int_0^4 (x^2+2)\,dx \qquad f(x) = x^2+2$



a) $L_4 = 1\left[f(0) + f(1) + f(2) + f(3)\right]$
$= 2 + 3 + 6 + 11$

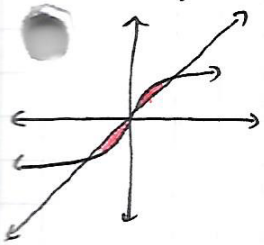
b) $R_4 = 1\left[f(4) + f(3) + f(2) + f(1)\right]$
$= 18 + 11 + 6 + 3$

c) $T_4 = \frac{1}{2}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + f(4)\right]$
$= \frac{1}{2}\left[2 + 2(3) + 2(6) + 2(11) + 18\right]$

d) $M_4 = 1\left[f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2}) + f(\frac{7}{2})\right]$
$= \frac{1}{4}+2 + \frac{9}{4}+2 + \frac{25}{4}+2 + \frac{49}{4}+2$

e) $\left[\frac{1}{3}x^3 + 2x + C\right]\Big|_0^4 = \boxed{\frac{64}{3} + 8}$

3) $y = \sqrt[3]{x} \qquad y = x$



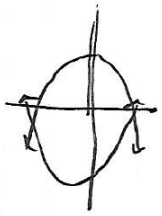
$\text{Area} = 2\int_0^1 \left[x^{1/3} - x\right]dx$

$= 2\left[\frac{3}{4}x^{4/3} - \frac{1}{2}x^2\right]_0^1$

$= 2\left[\frac{3}{4} - \frac{1}{2}\right]$

$= 2\left[\frac{1}{4}\right] = \boxed{\frac{1}{2}}$

4) $y = 25 - x^2 \qquad y = x^2 - 25$



$25 - x^2 = x^2 - 25$

$50 = 2x^2$

$x = \pm 5$

$\text{Area} = 2\int_0^5 \left[25 - x^2 - (x^2 - 25)\right]dx$

$= 2\int_0^5 \left[50 - 2x^2\right]dx$

$= 2\left[50x - \frac{2}{3}x^3\right]\Big|_0^5$

$= \boxed{2\left[250 - \frac{2}{3}(125)\right]}$

5) $f(x) = \begin{cases} kx - 1, & x < 2 \\ kx^2, & x \geq 2 \end{cases}$

$\lim_{x\to 2^-} f(x) = 2k - 1$

$\lim_{x\to 2^+} f(x) = 4k$

$2k - 1 = 4k$

$-1 = 2k$

$\boxed{k = -\frac{1}{2}}$

6) $y = x^2; \quad y = 2x + 3$

$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

$\text{Area} = \int_{-1}^3 \left[2x + 3 - x^2\right]dx$

$= \left[x^2 + 3x - \frac{1}{3}x^3 + C\right]\Big|_{-1}^3$

$= \boxed{\left[9 + 9 - 9\right] - \left[1 - 3 + \frac{1}{3}\right]}$

7) $y = \sin 3x$

$$\int_0^{\pi/3} \sin 3x \, dx$$

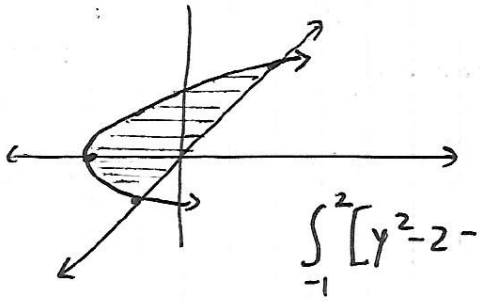
$$\left[-\frac{1}{3}\cos 3x + C\right]\Big|_0^{\pi/3}$$

$$\left[-\frac{1}{3}\cos \pi\right] - \left[-\frac{1}{3}\cos 0\right]$$

$$\frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

8) $x + 2 = y^2$ $\quad y = x$ $\quad x = y$ (dy)

$x = y^2 - 2$



$$\int_{-1}^{2} [y^2 - 2 - y]\, dy$$

9)

$$f(x) = \begin{cases} 3x^2 + 2, & x \geq -1 \\ -cx + 5, & x < -1 \end{cases}$$

$\lim_{x \to -1^-} f(x) = 5$ $\quad \lim_{x \to -1^+} f(x) = c + 5$

$$-c + 5 = 5$$

$$\boxed{c = 0}$$

10) $\frac{1}{\pi/4} \int_0^{\pi/4} \sec^2 x \, dx$

$$\frac{4}{\pi}\left[\tan x + C\right]\Big|_0^{\pi/4}$$

$$\frac{4}{\pi}\left[\tan \frac{\pi}{4} - \tan 0\right]$$

$$\frac{4}{\pi}\left[1 - 0\right]$$

$$\boxed{\frac{4}{\pi}}$$